Pressure drop for foam flow through pipes

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Although foams are two-phase materials, conventional two-phase flow calculation methods produce results very seriously in error if applied to foams. The reason is that the flow of a foam is dominated by the properties of a thin boundary region, which produce the effect of slip; the bulk properties, which are used in conventional methods, are less important to overall flow behavior. The most important parameter controlling the flow of foam in a pipe is the slip-layer thickness, which may be estimated from the average bubble diameter and expansion ratio. A relatively simple model with three fixed parameters and a slip-layer thickness calculated from the average bubble size can predict the pressure drops for many flows to within a factor of 2 in most cases. Improving the performance of this model in any particular situation may be possible with careful selection of the parameters. Within the accuracy of this model, effects of pressure variation on flowing foams can be allowed for satisfactorily by using averages of inlet and outlet properties and assuming isothermal expansion of bubbles.

Keywords: foam; pressure drop; friction; compressibility; pipe flow

Introduction

Foams may occur when a liquid and a gas are mixed intimately in the presence of a surface active agent. They are two-phase materials, and in some situations, whether a particular two-phase material is in the form of a foam may not be obvious. The distinction is, however, vital in situations in which the foam is likely to flow. The flow properties of a foam are very different from those of "conventional" two-phase fluids.

Foams consist of gas bubbles dispersed in liquid, but the gas normally comprises most of the volume. Thus the structure is relatively rigidly held together by surface tension effects, and the air-phase and liquid-phase velocities are the same. Foams may therefore be considered as homogeneous non-Newtonian fluids[1], so long as the flow scale is much larger than bubble size.

Two-phase correlations

Numerous standard correlations are used to estimate the pressure drop of two-phase fluids in pipe flow. One, which is widely used, is the Lockhart–Martinelli correlation and various modifications and extensions of it (see, for example, Chisholm[2]). We applied this correlation to a foam for comparison with experimental results. The experimental arrangements are described later in this article; here we consider a run in which an air–water foam of expansion ratio 8 flowed through a pipe 4 m long and 6.35 mm in diameter at a flow rate of 1.7 L/min.

The water and air superficial velocities are 0.11 m/s and 0.78 m/s, respectively, and the Reynolds numbers are 711 and 325. Both are therefore in the laminar region, and the pressure drops can be estimated at 354 Pa and 45 Pa, respectively.

The Lockhart–Martinelli parameter X^2 is thus 7.88 and the C-coefficient method for viscous–viscous flow gives a two-phase pressure drop of 1,030 Pa.

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The experimental pressure drop for foam flow was 94.5 kPa, a factor of 92 higher. This result is by no means unusual; in a program of about 400 experimental runs, the average ratio between measured and predicted pressure drops was 188, and the lowest was about 10. Figure 1 shows some typical results for this ratio, plotted against mean shear rate ($8Q/\pi D^3$ evaluated at outlet). Although some systematic variations are visible, clearly any prediction based on conventional two-phase methods is likely to be greatly in error.

Foam rheology

To understand why two-phase methods cannot be used for foams, we have to look at how a foam actually flows. The basic mechanisms are not fully understood, but there is general agreement about overall behavior (Heller and Kuntamukkula[3]). Foams are highly non-Newtonian, possessing a yield stress and a nonlinear shear characteristic. Near a solid surface, bubble migration leads to a liquid-rich layer, which gives the effect of slip between the foam and the wall. This layer may be idealized as a lubricating layer of pure liquid separating the foam from the surface.

The yield and shear characteristics can be explained qualitatively in terms of the interactions of the interbubble lamellae (e.g., Kraynik and Hansen[4]). The dominant parameters are the surface tension of the liquid and bubble size. However, in practice these properties cannot be predicted with any certainty and must be obtained experimentally (which is by no means straightforward). The effective viscosity of a foam, once slip effects have been removed, is usually several hundred times the viscosity of the base liquid.

Pipe flow

In pipe flow if the wall shear stress is below the yield stress, the foam cannot deform continuously (although there may be elastic deformation, not considered in this model) and will flow

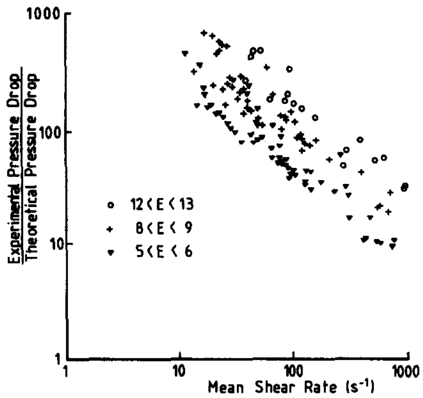

Figure 1 Comparison of predicted pressure drops using the Lockhart–Martinelli correlation with experimental pressure drops

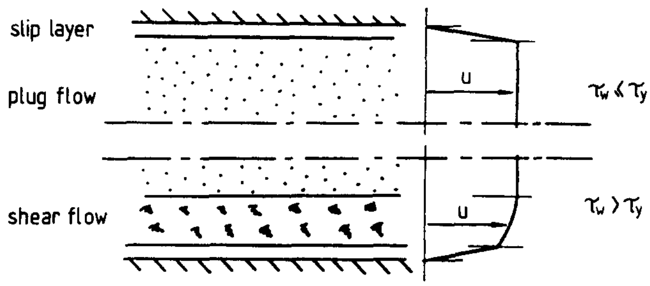

Figure 2 Velocity profile in pipe flow

as a constant-velocity plug lubricated by the wall slip layers. If the wall shear stress is above the yield stress, there will be shear in the foam as well as in the slip layer. Because the shear stress on the centerline must be zero, however, there will always be a central plug region. These situations are sketched in Figure 2; both occur within the range of foam flows commonly experienced.

The slip-layer thickness is on the order of tens of micrometers for typical foams. This small size, combined with the high value of the viscosity of the foam, means that a large proportion of the velocity variation in pipe flow occurs across the slip layer even in cases where the foam is shearing. The assumption of plug flow may therefore give a good first approximation even when the wall shear stress is above the yield stress.

Theory for plug flow

The shear stress across the slip layer will be virtually constant. We may therefore write

$$\tau_w = \frac{\mu u}{\delta} \tag{1}$$

where

τ_w is the wall shear stress;
μ is the viscosity of the base liquid;
u is the mean velocity; and
δ is the slip layer thickness.

The pressure drop may thus be estimated, provided that a value for slip-layer thickness is available.

We would expect the slip-layer thickness to be similar to the interbubble lamella thickness, which is related to the expansion ratio and the average bubble size:

$$e = \frac{2(\text{Liquid volume})}{\text{Bubble surface area}}$$

or

$$\frac{e}{d} = \frac{2a_v d_s}{a_s d(E-1)}$$

where

e is the lamella thickness;
d is the mean bubble diameter;
a_v is the bubble volume shape factor;
a_s is the bubble area shape factor;
d_s is the bubble surface/volume mean diameter; and
E is the foam expansion ratio.

Using values of δ from Figure 7 of Calvert and Nezhati,[5] we obtain the least-squares best fit:

$$\frac{\delta}{d} = \frac{2}{3(E-1)} \tag{2}$$

which is shown in Figure 3.

Assuming that an expression of this form is applicable to other situations, we can estimate pressure drop in plug flow from a knowledge of foam geometry (expansion ratio and bubble size) and liquid viscosity only.

In situations where pressure varies significantly along the pipe, we may modify Equations 1 and 2, making the assumption that the bubble expansion is isothermal. (Because of the relatively high specific heat of the liquid component, this is a very good approximation. We also ignore the excess internal pressure in the bubble, which arises from surface tension.) We assume that the initial bubble size is fixed by the foam generator. As the foam pressure falls, the bubble diameter will increase as the cube root of pressure. The increase in the expansion ratio is given by

$$p(E-1) = \text{Constant}$$

Therefore, from Equation 2, the slip-layer thickness varies as the two-thirds power of pressure.

The velocity will rise in proportion to the expansion ratio.

Notation

a_v, a_s	Bubble shape factors
D	Pipe diameter
d	Bubble diameter
E	Expansion ratio, (foam volume)/(liquid volume)
e	Lamella thickness
I, I_o	Terms in Equation 3
k	Consistency parameter in Calvert–Nezhati model
L	Pipe length
n	Flow index in Calvert–Nezhati model
P_o, P_i	Outlet and inlet pressure (absolute)
Q	Foam flow rate
u	Velocity
X	Lockhart–Martinelli parameter

Greek symbols

δ	Slip-layer thickness
τ_y	Yield stress
τ_w	Wall shear stress
μ	Viscosity of base liquid

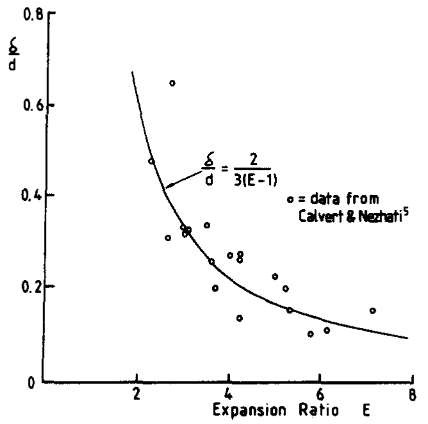

Figure 3 Variation of slip-layer thickness with expansion ratio

The wall shear stress may thus be calculated at any point where the pressure is known. As the wall shear stress is proportional to the pressure gradient, we may integrate the relation to find the total pressure drop.

In practice, solving for the flow rate at a specified pressure drop is easier. The exit velocity is

$$u=\frac{DdP_o}{6L\mu(E-1)}\times I \tag{3}$$

and

$$I=3E\left(\frac{P_i}{P_o}\right)^{1/3}\int_1^{P_i/P_o}\frac{(P/P_o)^{5/3}}{(P/P_o)+(E-1)}\,d(P/P_o)$$

where

- P_i is the inlet pressure;
- P_o is the exit pressure;
- D is the pipe diameter;
- L is the pipe length;
- d is the inlet (generator) bubble diameter;
- μ is the slip layer viscosity; and
- E is the exit expansion ratio.

The integral term I is a function of pressure ratio and expansion ratio only and can be evaluated readily for any desired values. The corresponding term in the absence of compressibility is

$$I_o=\left(\frac{P_i}{P_o}-1\right)$$

The ratio I/I_o is therefore a measure of the size of the compressibility effect. It is a measure of the increase in flow rate resulting from compressibility for a given pressure ratio and exit expansion ratio. This ratio is shown in Figure 4 for a range of pressure ratios and expansion ratios. As you can see, the ratio is relatively insensitive to the expansion ratio and roughly equal to the pressure ratio.

An alternative approach is to use the average of the input and output values of pressure and the expansion ratio to calculate an average velocity and then adjust this result to the exit pressure. This approach is considerably simpler computationally and gives values within about 10% of those in Figure 4.

Nonplug flow

If the flow is not a plug flow, a more complete rheological model is needed. Calvert and Nezhati's model[1] (the C–N model) assumes that the foam (after allowing for slip) may be represented by a yield stress τ_y, below which there is no shear. Above the yield stress, the flow shears according to a power law with consistency k and flow behavior index n. The flow rate–wall shear stress equations for this model are given in the Appendix.

The C–N model requires four parameters: k, n, τ_y, and δ. However, Calvert and Nezhati noted that the values of k and n were fairly stable from one set of results to another and suggested the use of average values of 2.5 and 0.4, respectively. Also, the equations in the Appendix are relatively insensitive to τ_y, because at low values of τ_w very little of the flow arises from the shearing part, whereas at high values errors in τ_y will be unimportant. A typical value of about 1 N/m^2 may therefore give acceptable results. In this situation also, knowledge of δ, which can be obtained from Equation 2, is all that is required to calculate the pressure drop. In this case, the equations are not amenable to the integration carried out in Equation 3, and some form of averaging must be used to allow for compressibility.

Experimental procedures

Approximately 400 experimental runs were made in which the pressure drop through a pipe was measured. The pipes were straight runs of polyethylene tubing between 4 m and 10 m long, with nominal diameters ranging from 3.175 mm to 25.4 mm. Pressure was measured at the inlet end, the exit being at atmospheric pressure.

Flow rate and expansion ratio were measured at the exit. To avoid potential problems from throttling the flow, the flow rate was controlled by a bypass system. Foam flow rates ranged between 0.1 L/min and 13.4 L/min and expansion ratios from 3.3 to 18.8 (all measured at atmospheric pressure).

The foaming mixture was a 3% aqueous solution of BP By-Prox general purpose detergent, expanded with air. The foam generator was based on a packed bed of grade 4 (0.87-mm–1.275-mm) glass ballotini.

Pressure drops in the pipes ranged up to approximately 160 kPa and thus are not negligible compared to atmospheric pressure. This result means that the bubble size, and consequently the expansion ratio and foam velocity, will usually vary significantly along the pipe.

Results

Because of the number of variables and runs, not all the detailed results are presented here. Figures 5 and 6 show typical sets of results for pressure drop in particular geometries, as a function

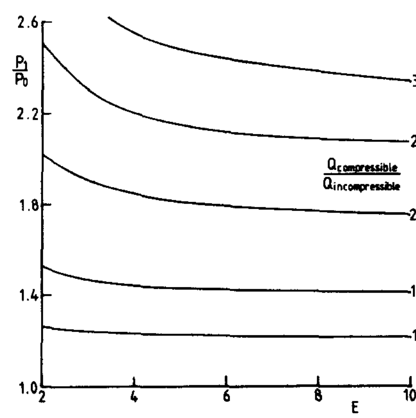

Figure 4 Effect of compressibility on flow rate for plug flow

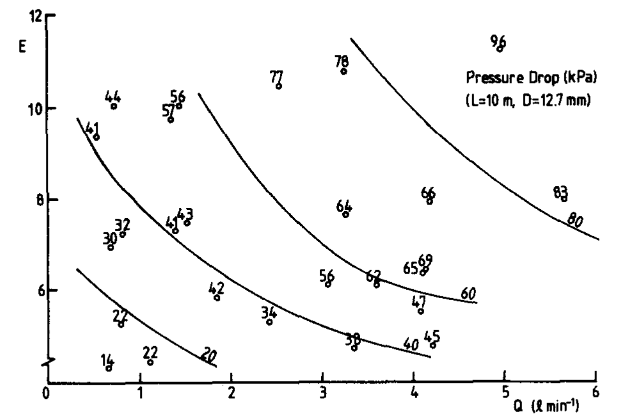

Figure 5 Pressure drop as a function of the flow rate and expansion ratio (10-m length, 12.7-mm diameter)

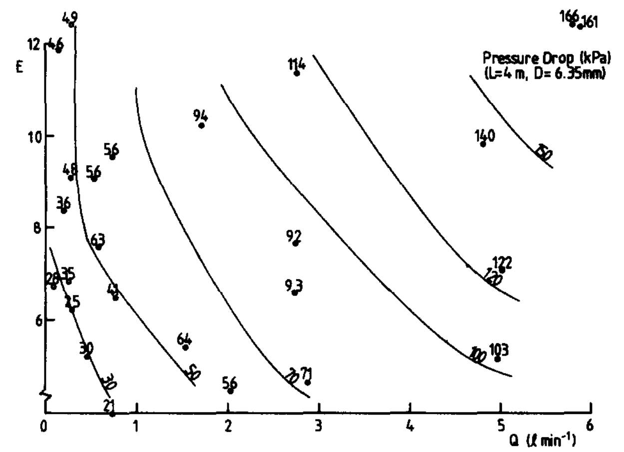

Figure 6 Pressure drop as a function of the flow rate and expansion ratio (4-m length, 6.35-mm diameter)

of the expansion ratio and foam flow rate. (The lines of constant pressure drop were sketched by eye and should not be taken as particularly accurate.) In these cases, the pressure drop rises with both the flow rate and expansion ratio, which holds for all geometries measured. Other results are reported later in this article.

Comparison with experiment

Plug-flow model

Equations 1 and 2 suggest that the pressure drop in plug flow should be proportional to velocity or flow rate (neglecting compressibility effects). Figure 7 shows experimental results for two different exit expansion ratios in the same pipe (which was 4 m long and 6.35 mm in diameter). The pressure drop is clearly not proportional to flow rate. Figure 7 also shows the curve predicted by Equation 3, which allows for compressibility, for an expansion ratio of 10 and $\delta = 20\,\mu m$. The order of magnitude is correct, although the fit is far from good.

The only "free" parameter in Equation 3 is δ. Variation of δ does not appreciably change the shape of the curve, so improving the fit significantly is not possible. Furthermore, Equation 3 is fairly insensitive to E (note that at constant δ—and using Equation 2—E appears only in the integral term I) and, in fact, predicts increasing flow with increasing E, contrary to experimental results. We must conclude therefore that even with allowance for pressure change, the plug-flow model is inadequate for these data.

Shear-flow model

The final line in Figure 7 is a curve predicted by the C–N model. The parameters for the model were selected as convenient round numbers: $k = 2.5$ (SI units), $n = 0.4$, $\tau_y = 1.0\,N\,m^{-2}$ and $\delta = 20\,\mu m$. Compressibility effects were neglected. This curve is remarkably similar to the experimental curves, considering that no attempt was made to optimize it (because of the highly nonlinear nature of the model, optimization is not easy). This result suggests that applicability of the C–N model may extend beyond the conditions for which it was proposed (the foam material, foam generator principle, flow rates, and pipe dimensions were all different from those reported here, which cover a much wider range of conditions).

The value of yield stress used in the C–N model is typical of many foams and is well below the minimum wall shear stress at any of the points on Figure 7 (about $8\,N\,m^{-2}$). The foam is therefore shearing at all points. The curve diverges from the plug-flow curve at a pressure drop of about 40 kPa, which corresponds to a τ_w/τ_y ratio of about 15. The proportion of the flow resulting from shear, however, is small; it does not reach 50% until τ_w/τ_y is about 50. Figure 8 shows the proportion of flow resulting from shear as a function of δ/D and τ_w/τ_y for $k = 2.5$, $n = 0.4$, and $\tau_y = 1\,N\,m^{-2}$. (The equations in the Appendix may be used to generate similar charts for other sets of parameters.)

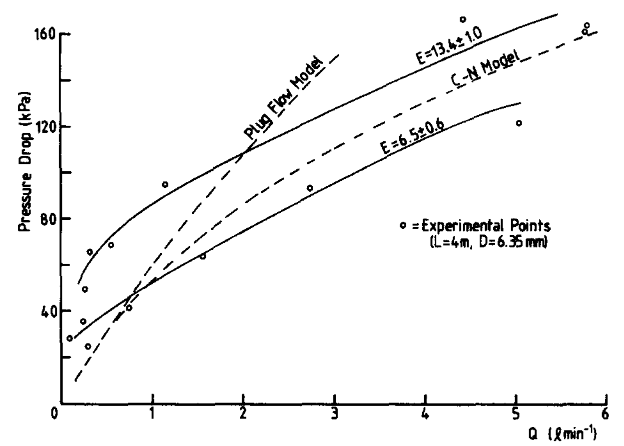

Figure 7 Comparison of pressure drop results and theoretical models

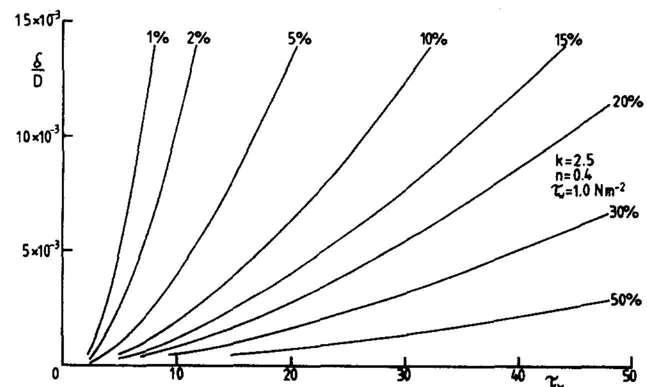

Figure 8 Proportion of flow resulting from foam shear ($k = 2.5$, $n = 0.4$, $\tau_y = 1.0\,N\,m^{-2}$)

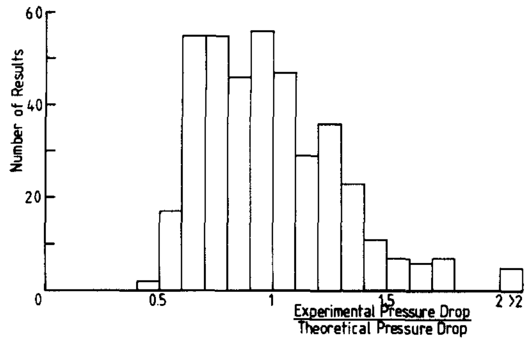

Figure 9 Histogram of results predicted by the Calvert–Nezhati model

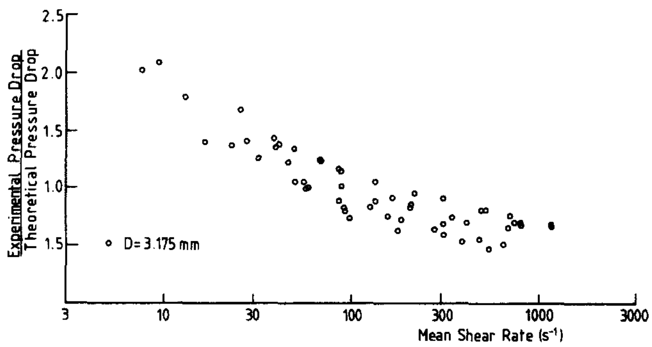

Figure 10 Comparison of predicted pressure drops, using the Calvert–Nezhati model, and experimental pressure drops (pipe diameter, 3.175 mm)

Encouraged, we used the C–N model with three fixed parameters (as in the preceding paragraph) to predict the pressure drop for all data points. The only free parameter is δ, which we estimated from bubble size and the expansion ratio, using Equation 2. For the present data, we selected a foam generator bubble diameter of $83\,\mu m$ by trial and error to give an average ratio of experimental to calculated pressure drop of 1.0 (as no measurements of bubble size were available). We used these values to estimate slip-layer thickness and used Equations A1 and A2, iteratively, to obtain the pressure drop for the experimental value of the flow rate. We allowed for compressibility by averaging the values of the expansion ratio and absolute pressure at inlet and exit, using them to calculate an average flow rate and then adjusted this to exit conditions.

The results were remarkably good—98% of the results were correct to within a factor of 2 and 75% to within a factor of 1.5. Figure 9 is a histogram of the ratios of experimental to theoretical pressure drops for all data points. Figure 10 shows the results for the 3.175-mm diameter pipe, plotted as a function of mean shear rate.

The histogram shows a reasonably symmetrical distribution, suggesting that the few points lying outside the factor-of-2 range are in some way anomalous. Figure 10 shows the ratio falling from about 1.8 at low shear rates to about 0.7 at high shear rates. The pattern is similar, though less marked, for other sets of data. The pressure drop is thus being overestimated at high shear rates relative to low shear rates.

Adjustment of the four parameter values should improve the overall fit. This adjustment was not attempted in any detail because of the problems of nonlinear optimization on data with a broad scatter. Some numerical experiments suggest that slight improvement can be obtained by allowing the yield stress to vary with the expansion ratio and bubble size, in the manner shown in Calvert and Nezhati (Figure 8).[5] The improvement is not enough, however, to justify introduction of two more arbitrary constants without a reasonable theoretical basis. Attempts (e.g., Princen,[6] and Kraynik,[7]) have been made to relate yield stress, expansion ratio, slip-layer thickness, and bubble size theoretically. These investigators usually used a two-dimensional (2-D) model and restricted themselves to expansion ratios above about 10. The general trends they predict are not very consistent with those of the experimental data mentioned above.

Conclusions

Conventional two-phase flow calculation methods for pressure drop in pipes produce seriously erroneous results if applied to foams. The most important parameter controlling the flow of foam in a pipe is the slip-layer thickness, which may be estimated from the average bubble diameter and expansion ratio.

A plug-flow model may be used to estimate pressure drop at low wall shear stresses. The model is inaccurate (although of the right order of magnitude) if the wall shear stress exceeds about 15 times the foam yield stress.

The Calvert–Nezhati (C–N) model with three fixed parameters—and a slip-layer thickness calculated from the average bubble size—can predict the pressure drops for the type of flows considered here to within a factor of 2 in most cases. Improving the performance of this model in any particular situation by careful selection of the parameters may be possible.

Within the accuracy of prediction of the C–N model, effects of pressure on flowing foams can be allowed for satisfactorily by using averages of inlet and outlet properties and assuming isothermal expansion of bubbles.

Acknowledgments

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Appendix: Equations for the Calvert–Nezhati model

Flow model

In bulk foam:

$$\tau = \tau_y + k\gamma^n \qquad (\tau > \tau_y)$$

$$\gamma = 0 \qquad (\tau < \tau_y)$$

where

τ = shear stress;
τ_y = yield stress;
k = consistency;
γ = shear rate;
n = flow behavior index; and
δ = slip-layer thickness.

Near a solid surface:

The slip layer has thickness δ and has the viscosity of the base liquid.

Pipe flow

$Q=Q_s+Q_f$

where

Q = foam volumetric flow rate;
Q_s = flow rate because of slip; and
Q_f = flow rate because of foam shear.

$$Q_s=\frac{\pi\delta D^2\tau_w}{4\mu} \tag{A1}$$

where

D = pipe diameter;
τ_w = wall shear stress; and
μ = slip-layer viscosity.

$Q_f=0 \qquad (\tau<\tau_y)$

$$\frac{8Q_f}{\pi D^3}=\left(\frac{\tau_w}{k}Y\right)^{1/n}\left(\frac{n}{(n+1)}Y\right)\left[1-\frac{2n}{(2n+1)}Y\left(1-\frac{n}{(3n+1)}Y\right)\right] \qquad (\tau>\tau_y) \tag{A2}$$

where

$Y=(1-\tau_y/\tau_w)$.